

# Interference mechanism of magnetoresistance in variable range hopping conduction: the effect of paramagnetic electron spins and continuous spectrum of scatterer energies.

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## Abstract

Despite the fact that the problem of interference mechanism of magnetoresistance in semiconductors with hopping conductivity was widely discussed, most of existing studies were focused on the model of spinless electrons. This model can be justified only when all electron spins are frozen. However there is always an admixture of free spins in the semiconductor.

This study presents the theory of interference contribution to magnetoresistance that explicitly includes effects of both frozen and free electron spins. We consider the cases of small and large number of scatterers in the hopping event. For the case of large number of scatterers the approach is used that takes into account the dispersion of the scatterer energies. We compare our results with existing experimental data.

PACS numbers: 72.20.Ee, 75.47.-m

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## I. INTRODUCTION

At low temperatures the conductivity in semiconductors is supported by carriers hopping between localized states on the impurities. At low enough temperatures the characteristic act of hopping occurs not between the neighboring impurities but between the closest impurities in the thin energy strip around the Fermi level. This phenomenon is known as variable range hopping conductivity and is characterized by the well-known temperature dependence of resistance:

$$R(T) \propto \exp\left(\frac{T}{T_0}\right)^\alpha, \quad (1)$$

where  $T_0$  is a constant and  $\alpha$  is less than unity. When constant density of states  $g(E)$  is discussed, the exponent  $\alpha$  is equal to  $1/(d+1)$ , where  $d$  is the system dimension. However when the conduction strip becomes narrower and the Coulomb gap becomes important, then  $\alpha = 1/2$  for any system dimension. The variable range hopping conductivity is, in particular, discussed in details in [1].

The problem of magnetoresistance in the semiconductors has been actual since 1950-th [2]. Then it was understood that at strong magnetic fields the magnetoresistance is positive and can be exponentially strong. This magnetoresistance is due to electron wavefunction shrinkage in the magnetic field.

However it has appeared that at weaker fields the magnetoresistance becomes negative. This phenomenon has been first described by Nguen, Shklovskii and Spivak [3] (for detailed review see [4]). It is related to the interference. During the hop the tunneling carrier suffers under-barrier scattering on the impurities that are outside of the conduction strip. So the resulting hopping amplitude becomes the sum of different tunneling paths, which interfere with each other.

It is important that the hopping conductivity is controlled by percolation and, thus, by the largest resistors (formed by the pairs of hopping sites) that still should be included into the percolation network. This fact strongly emphasizes a role of destructive interference which can practically cut the percolation path and leads to an increase of the resistance. The magnetic field suppresses the interference and leads to negative magnetoresistance.

The negative magnetoresistance appears to be linear (on the magnetic field) in the relatively wide range of weak fields, so it dominates over quadratic positive wave-shrinkage magnetoresistance at weak magnetic field.

The theory of interference magnetoresistance [4] is based on the model of spinless electrons. This model can be justified when all electron spins are frozen, for example, by the exchange interaction. In [4] it is stated that there is no interference effect on the magnetoresistance when all electron spins are free. However this statement lacks the solid theoretical proof. Also, no discussion of the systems in which some spins are frozen and some are free is given. Therefore the question remains "how many free spins one should have to suppress the interference magnetoresistance?".

In [5] it is stated that the admixture of free spins leads to additional positive magnetoresistance due to spin alignment in the magnetic field. However no detailed discussion of this magnetoresistance mechanism is given. For example its detailed temperature dependence remained unknown.

In the studies [3, 4] authors consider the system with a large number of scatterers involved in a hopping event. In our study we denote this situation as the case of long hops. For this model only qualitative results are obtained in [3, 4]. Later it was shown [6, 7] that in many realistic systems the characteristic number of scatterers is small. Most hopping events occur without scattering, and to consider interference magnetoresistance one can take into account only hopping events including a single scatterer. We denote this situation as the case of short hops. For this problem of short hops (and spinless electrons) the papers [6, 7] give quantitative estimates of the magnetoresistance.

There were several attempts [8–12] to develop a quantitative theory for long hops. However, to our opinion, no one of this works pays a sufficient attention to the variance of scatter energies.

Also we state that the conventional theory of interference contribution to magnetoresistance is not sufficient to describe existing experimental data. While semiconductors with hopping conductivity usually exhibit a combination of negative linear and positive quadratic magnetoresistance (this fact is in accordance with conventional magnetoresistance theory), the temperature dependence of negative and positive magnetoresistance differs sometimes from predicted by existing theories [13–19]. In [13, 15, 19] this difference is attributed to spin effects. In [15, 19] the spin alignment magnetoresistance (suggested in [5]) is invoked to describe experimental observations. However no rigorous theory of this magnetoresistance (and any consistent discussion of free spins effect on the magnetoresistance) is presented.

The goal of the present study is to develop theoretical approach to the interference con-

tribution to magnetoresistance that explicitly takes into account an admixture of free spins. We consider two general problems. The first is to what extent the interference magnetoresistance is suppressed when spins become free. The second is related to derivation of the expression for spin alignment magnetoresistance that would give the detailed dependence of this magnetoresistance on the system parameters including magnetic field, temperature and the weight of the free spins.

In our work we consider both long and short hops. For long hops (large scatterer numbers) we use an approach that takes into account the scatterer energy variance. Our approach becomes rigorous for exponentially broad distribution of scatterers energies. However we show with numerical computations that it is applicable to realistic energy variances.

The plan of this study is as follows. In section II we discuss the basics of interference magnetoresistance mechanism. In section III we present theoretical analysis for the short hop case. In section IV we discuss the statistics of tunneling path amplitudes in the long hop limit. In section V we use this statistics to get expression for magnetoresistance in this case. In section VI we compare our results with existing experimental data and in section VII we present the final discussion of our results.

## II. INTERFERENCE MECHANISM OF MAGNETORESISTANCE

It is convenient to describe hopping conductivity in terms of random resistance network first proposed by Miller and Abrahams [20]. In terms of this model any pair of impurities (denoting as 1 and 2) corresponds to a resistor with resistivity

$$R_{12} \propto \Gamma_{12}^{-1}, \quad \Gamma_{12} \propto \left\langle \left| \tilde{I}_{12} \right|^2 n_1 (1 - n_2) N_{ph} \right\rangle. \quad (2)$$

Here  $\Gamma_{12}$  is the hopping rate,  $n_1$  and  $n_2$  are the occupation numbers of the first and the second impurity states, correspondingly.  $\tilde{I}_{12} \propto \exp(-r_{12}/a)$  is the energy overlap integral between states 1 and 2,  $r_{12}$  is the distance between impurities and  $a$  is the localization length.  $N_{ph}$  is the probability to find a phonon for a hop (one can take  $N_{ph} = 1$  for hops with emission of a phonon). Finally, angle brackets mean time averaging.

The terms  $n_1$ ,  $(1 - n_2)$  and  $N_{ph}$  specify the dependence of resistor  $R_{12}$  on impurity energies. We will not discuss this part of (2). What we are interested in is the overlap integral  $\tilde{I}_{12}$ .

When there are no impurities beside 1 and 2, the overlap integral can be easily evaluated [1]

$$\tilde{I}_{12} = I_0 \left( \frac{r_{12}}{a} \right)^\beta \exp(-r_{12}/a). \quad (3)$$

Here  $I_0$  is the constant of the order of the Bohr energy,  $\beta$  is the pre-exponential factor that is determined by the impurity type. For shallow impurities in the bulk system (or in the  $\delta$ -doped layer)  $\beta = 1$ . For non-Coulomb impurities (like the deep impurities or doubly-occupied  $D^-$  or  $A^+$  centers)  $\beta = -1$ . In two-dimensional systems  $\beta$  is equal to 1/2 and -1/2 correspondingly for Coulomb and non-Coulomb impurities.

When there are intermediate scattering impurities between the hopping ones, the overlap integral becomes a sum of the different tunneling paths and expression (3) stands only for one of them (the one without scattering). The total overlap integral between impurities 1 and 2 can be expressed as

$$\tilde{I}_{12} = I_{12} + \sum_i \frac{I_{1i}I_{i2}}{E_i} + \sum_{ij} \frac{I_{1i}I_{ij}I_{j2}}{E_i E_j} + \dots \quad (4)$$

Here  $I_{ij}$  are defined by (3).  $E_i$  are the energy positions of scattering impurities (with respect to the Fermi level). We assumed that the corresponding energies are much larger than the energies of hopping impurities 1 and 2. Each summand in (4) corresponds to an amplitude of some tunneling path. Let us note that any tunneling path amplitude is  $\propto \exp(-l/a)$  where  $l$  is the length of the corresponding hopping segment. So the paths with large  $l$  can be neglected. Accordingly, we neglect all paths with backscattering and also include in our consideration only scattering impurities in some thin strip (in real space) around the line, connecting impurities 1 and 2. The width of this strip is estimated in [4] as  $\sqrt{r_{12}a}$ . However we will show that in the case of long hops it is sufficient to consider a thinner strip. So we have a finite number of paths  $2^N$  where  $N$  is the number of scatterers in the strip.

Let us also write the expression for the simplest (but important) case of one scattering impurity  $s$ . In this case

$$\Gamma_{12} \propto |J_1 + J_2|^2, \quad J_1 = I_{12}, \quad J_2 = \frac{I_{1s}I_{s2}}{E_s}. \quad (5)$$

Note that the expression (5) and the upper expressions do not take in account the electron spin. Consequently up to this moment we discussed the model of spinless electrons. As we have already mentioned, this model can be justified when electron spins on scattering

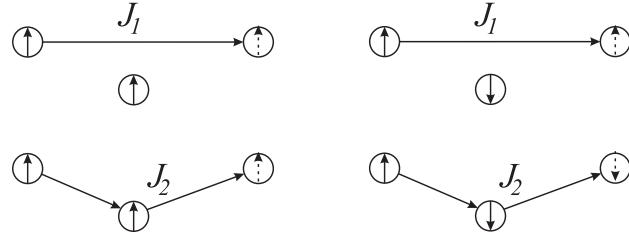


FIG. 1: Final results of direct and scattered tunneling path in different spin configuration.

impurities are frozen (for details see [4]). But now let us assume that the intermediate impurity is occupied and its electron spin is free.

The scattering on the occupied impurity is equivalent to the cotunneling of the initial electron to the scatterer and the electron from the scatterer to the destination impurity (figure 1). One can see that when spins of initial electron and electron on the scatterer have the same directions, the final result of direct tunneling  $J_1$  and the tunneling with scattering  $J_2$  are the same. However, when the directions of these spins are antiparallel, the final results of the trajectories  $J_1$  and  $J_2$  are different. So in this case the tunneling paths do not interfere.

As long as the spin on the scattering impurity is free, there is always a probability that the two spins have the same direction and a probability that they have antiparallel directions. Note that the expression (2) for  $\Gamma_{ij}$  contains time averaging. So for the free spin on the intermediate impurity one obtains

$$\Gamma_{12} \propto P_{\uparrow\uparrow} |J_1 + J_2|^2 + (1 - P_{\uparrow\uparrow}) (|J_1|^2 + |J_2|^2). \quad (6)$$

Here  $P_{\uparrow\uparrow}$  is the probability for two spins to have the same direction. Without the magnetic field  $P_{\uparrow\uparrow} = 1/2$  because both spin directions are equally probable. The external magnetic field aligns the spins leading to an increase of  $P_{\uparrow\uparrow}$ . The expression for probability  $P_{\uparrow\uparrow}$  in a magnetic field is as follows

$$P_{\uparrow\uparrow} = \frac{\cosh(2\mu H/T)}{2 \cosh^2(\mu H/T)} = \frac{1}{2} + \frac{1}{2} \left( \frac{\mu H}{T} \right)^2 + O(H^4). \quad (7)$$

Here  $\mu$  is the magnetic moment of a localized electron,  $H$  is the external magnetic field and  $T$  is temperature expressed in energy units.

In the expression (6) we have implicitly assumed that the spin states change with time faster than the hopping events occur or at least faster than Miller-Abrahams network is

stabilized. To explain this assumption we note that the spin states at least can not change significantly slower than the hopping rate because the hopping in the antiparallel spin combination can exchange the spins. Also, other mechanisms of spin diffusion or relaxation can exist, for example, related to the exchange of spin directions between neighbor impurities with free spin. We also note that our expression (6) agrees with expression (14) from [5] (that describes spin alignment magnetoresistance).

There is another assumption that we will rely upon. Namely, we will make use of the fact that the frozen spins form the so called Bhatt-Lee phase [21]. In this state the frozen spins are in a singlet state and thus have no preferred direction. So the frozen spins scatter all tunneling electrons in the same way (independently of their spin direction). If by some reason the frozen spins form spin glass state, our considerations can not be applied.

One can see that there is a dependence of  $\Gamma_{12}$  on magnetic field even if the intermediate impurity has a free spin. First, in magnetic field a phase difference between tunneling paths exist. It can be expressed as  $J_2 \rightarrow J_2 e^{i\varphi}$ ,  $\varphi = HS/\Phi_0$ , where  $S$  is the area between the tunneling paths,  $\Phi_0$  is the magnetic flux quantum. Secondly, the probability  $P_{\uparrow\uparrow}$  depends on the magnetic field (see Eq. (7)). According to the logarithmic averaging procedure ([4]), one can write the following expression for the contribution of scattering on free spins states to the magnetoresistance:

$$\ln \frac{R(H)}{R(0)} = - \left\langle \ln \frac{P_{\uparrow\uparrow} |J_1 + J_2 e^{i\varphi}|^2 + (1 - P_{\uparrow\uparrow}) (|J_1|^2 + |J_2|^2)}{|J_1 + J_2|^2 / 2 + (|J_1|^2 + |J_2|^2) / 2} \right\rangle_{free}. \quad (8)$$

Here the angle brackets with index "free" mean the averaging over critical resistors with free spin on the scatterer (note that in short hop limit we have only one scatterer). There is also a contribution from resistors with unoccupied intermediate impurity and from resistors involving a scattering impurity with frozen electron spin. These contributions have a similar form:

$$\ln \frac{R(H)}{R(0)} = - \left\langle \ln \frac{|J_1 + J_2 e^{i\varphi}|^2}{|J_1 + J_2|^2} \right\rangle_{frozen}. \quad (9)$$

Note that angle brackets with index "frozen" mean not only the averaging over scatterers with an electron with frozen spin but also over the unoccupied scatterers. The contribution of these two scatterer types have the same form although the effects of these scatter types are different due to the sign of energy  $E_s$ .

Now we generalize the discussed approach for the situation when a resistor includes  $N$  scattering impurities and  $M$  of them are occupied by electrons with free spins. When a resis-

tor contains one free spin, there are four possible initial spin configurations (corresponding to up and down projection of initial spin and free spin on the scatterer). Similarly,  $M$  free spins in a resistor lead to  $2^{M+1}$  possible initial spin configurations. Also, in a resistor with one free spin one has no more than two possible tunneling results (final states of the tunneling) for any initial configuration — with and without exchange of spin projections. In our resistor one has no more than  $2^M$  tunneling results for any spin configuration. These results can be described by the combination of free spins that changes the spin projection during the process of hopping. Note that some results are impossible in specific spin configurations (for example for one intermediate free spin there is only one tunneling result for spin configurations with parallel spin projections).

Accordingly, for any spin configuration  $2^N$  tunneling amplitudes can be separated into  $2^M$  groups corresponding to different tunneling results (some groups can contain no tunneling amplitudes in some initial spin configurations). The tunneling paths that are inside one group interfere (because they lead to the same final state in the given initial configuration of free spins). Other tunneling paths do not interfere. One should average  $\Gamma_{12}$  over initial configurations:

$$\Gamma_{12} \propto \sum_{c=1}^{2^M} P_c(H) \sum_r \left| \sum_{k(c,r)} J_k e^{i\varphi_k} \right|^2. \quad (10)$$

Here index  $c$  enumerates possible spin configurations,  $P_c(H)$  is the probability of a given spin configuration in the magnetic field to exist. Index  $r$  enumerates the tunneling results.  $J_k$  are the amplitudes of the tunneling paths and  $\varphi_k$  are the corresponding phases. The sum over  $k$  is taken only over those tunneling paths that lead to result  $r$  in the spin configuration  $c$ . The probability  $P_c(H)$  can be expressed as

$$P_c(H) = \frac{\exp[(2N_{up}(c) - M - 1)\mu H/T]}{2^{M+1} \cosh^{M+1}(\mu H/T)}, \quad (11)$$

where  $N_{up}(c)$  is the number of free electron spins (including the spin of tunneling electron) that are aligned along the magnetic field in the configuration  $c$ .

The corresponding expression for magnetoresistance is

$$\frac{R(H)}{R(0)} = - \left\langle \ln \frac{\sum_{c=1}^{2^M} P_c(H) \sum_r \left| \sum_{k(c,r)} J_k e^{i\varphi_k} \right|^2}{\sum_{c=1}^{2^M} P_c(0) \sum_r \left| \sum_{k(c,r)} J_k \right|^2} \right\rangle. \quad (12)$$

Here the angle brackets mean averaging over critical resistors. We note that number  $M$ , possible spin configurations and tunneling results can be different for different critical resistors.

When there are no resistors with two or more scatterers, the expression (12) is reduced to (8) and (9). When all electron spins are frozen, it is reduced to the conventional expression for magnetoresistance from [4] (that corresponds to  $\Gamma_{12} \propto |\tilde{I}_{12}|^2$  with  $\tilde{I}_{12}$  defined in (4)).

The expression (12) is rather complex. However it allows numerical computation of magnetoresistance for a relatively large number of scatterers. The results of such computations will be presented in the section V of this study along with analytical analysis of the magnetoresistance.

### III. SHORT HOPS

In the case of short hops one can neglect resistors with more than one scatterer. Hopping events with no scatterer do not contribute to interference magnetoresistance so that one can consider only resistors with one scatterer. For those resistors it is possible to use expressions (8) and (9).

The magnetoresistance corresponding to short hop limit with no free electron spins (expression (9)) is known from [6, 7]. It can be shown that the averaging in (9) yields different results for the two areas. In the first area,  $A_1$ , the denominator is not very small  $|J_1 + J_2| > \varphi J_1$ . In this area one can expand the logarithm. It leads to a quadratic magnetoresistance. In the second area,  $A_2$ , where  $|J_1 + J_2| < \varphi J_1$ , the logarithm is large and can not be expanded. This area leads to the negative linear magnetoresistance that dominates in weak fields. For short hops the expression for this magnetoresistance is (see [6, 7])

$$\ln \frac{R(H)}{R(0)} \propto -r_h^{2+d/2} H, \quad (13)$$

where  $r_h$  is the mean hopping distance and  $d$  is the system dimensionality. The dependence on  $r_h$  controls the temperature dependence of the magnetoresistance as  $r_h \propto T^{-1/(d+1)}$  for the Mott-like hopping and  $r_h \propto T^{-1/2}$  for Efros-Shklovskii hopping over Coulomb gap states.

For the free electron spins the area  $A_2$  does not exist. Actually, the denominator in (8) is not less than  $3|J_1|^2/4$ . The denominator in (9) for resistors with unoccupied scatterers (that has constructive interference) is larger then  $|J_1|^2$ . Hence in weak magnetic field the

logarithm can always be expanded. This leads to the following expression for the weak field magnetoresistance expansion:

$$\ln \frac{R(H)}{R(0)} \approx - \left\langle \frac{(\frac{\mu_b g H}{T})^2 J_1 J_2 - \frac{1}{2} J_1 J_2 \varphi^2}{\frac{1}{2}(J_1 + J_2)^2 + \frac{1}{2}(J_1^2 + J_2^2)} \right\rangle_{occupied} + \left\langle \frac{J_1 J_2 \varphi^2}{(J_1 + J_2)^2} \right\rangle_{unoccupied}. \quad (14)$$

Here angle brackets with index "occupied" mean averaging over resistors with occupied scattering impurity (in this expression we consider all such impurities to have a free electron spin). Note that for this impurities  $J_1 J_2$  is negative. Angle brackets with index "unoccupied" corresponds to averaging over resistors with free intermediate impurity. For those resistors  $J_1 J_2$  is positive.

There are three contributions to magnetoresistance *due to interference* for free electron spins. First, there is one negative contribution. It comes from magnetic field suppression of destructive interference for the resistors with occupied scatterers. However, in contrast to the situation when electron spins are frozen, it is quadratic (not linear) in terms of the magnetic field and does not automatically dominate over other terms in a weak field limit. Also, it does not automatically dominate over the wave shrinkage magnetoresistance even for weak fields.

The second term appears from the suppression of interference in resistors with unoccupied scatterers and constructive interference. It will dominate the first term at least for semiconductors with large compensation  $K$  ( $1 - K \ll 1$ ), as the number of unoccupied scatterers in this semiconductors is larger than the number of occupied ones.

The third term in (14) is also positive and quadratic in terms of magnetic field

$$-\left\langle \frac{(\frac{\mu_b g H}{T})^2 J_1 J_2}{\frac{1}{2}(J_1 + J_2)^2 + \frac{1}{2}(J_1^2 + J_2^2)} \right\rangle_{occupied} \propto (nr_h^{(d+1)/2} a^{(d-1)/2}) \left( \frac{\mu_b g H}{T} \right)^2. \quad (15)$$

Here  $n$  is the impurity concentration,  $d$  is the system dimensionality. On the right-hand side of the equation we used the following approximation. We considered  $J_1$  and  $J_2$  to be of the same order (it is usually correct if the scatterer lies in the thin area between the hopping impurities [4]).  $(nr_h^{(d+1)/2} a^{(d-1)/2})$  is the probability to find a scatterer in this area.

The term (15) is related to the dependence of the probability  $P_{\uparrow\uparrow}$  on the magnetic field. It is the spin-ordering magnetoresistance first discussed in [5]. One can see that this term has very strong temperature dependence. For example, for a 2D system with hopping over the Coulomb gap states, this term depends on the temperature as  $T^{-11/4}$ . To compare, the temperature dependence of the wave-shrinkage magnetoresistance in this case is  $\propto T^{-3/2}$ .

Therefore at low temperature the spin-alignment magnetoresistance should become stronger than the wave-shrinkage one.

However, at low temperatures the system should correspond to the case of long hops (rather than of the short hops) and at some temperature electron spins should be controlled by the exchange interaction. However (as we show in section V) the spin alignment magnetoresistance remains in the long hop limit and has a strong temperature dependence.

Finally, one can consider the situation when some spins are frozen and some spins are free, that is  $0 < P_{free} < 1$ . The short hops theory of magnetoresistance can be easily generalized for this case. Naturally, each resistor contains either a free spin or a frozen one (if it involves an occupied scatterer). The general expression for magnetoresistance contains averaging over critical resistors. Accordingly the following equation can be used for shot hops

$$\ln \frac{R(H)}{R(0)} = P_{free} MR_{free} + (1 - P_{free}) MR_{frozen},$$

where  $MR_{free}$  is given by the expression (14) and  $MR_{frozen}$  is given by the expression (13). Naturally, when there is a significant number of frozen spins in the system ( $1 - P_{free}$  is not very small) the weak field magnetoresistance will be controlled by  $MR_{frozen}$  and will be negative and linear in terms of magnetic field. However, if one considers the quadratic term of the magnetoresistance expansion, then the term  $MR_{free}$  can also become important.

#### IV. LONG HOPS. STATISTICS OF TUNNELING PATHS

We address now the case of long hops, i.e. the situation when there is a large number of scatterers in a hopping process. As we have mentioned before, in this case there is exponentially large number of tunneling paths  $2^N$  (even without backscattering), where  $N$  is the number of intermediate impurities in the resistor. When there are no free spins, all paths interfere. The situation with free spins is more complex as it was discussed in section II.

In our opinion, the role of free spins in this case is even more important than it is for short hops. Indeed, for sufficiently long hops free spins are always present in the area between hopping sites. It is known that with decreasing temperature the spins of electrons that are localized on the impurities form the so-called Bhatt-Lee phase [21]. One of the properties of this phase is a relatively large number of free spins. The simplest estimate of

the number of free spins is based on the nearest-available-neighbor approach [22]. It leads to the logarithmic decrease of the free spin concentration with decreasing temperature. The hopping length grows with temperature according to a power law. So at low temperatures the number of scatterers with free spins in a hopping process is large.

In [4] the following arguments for the absence of interference magnetoresistance in the case of free electron spins has been proposed. It has been stated that in this case there are many tunneling paths that do not interfere with each other, making the strong destructive interference (that is responsible for linear negative magnetoresistance) very unlikely.

However, at sufficiently low temperatures each critical resistor has free spins and, therefore, there is always a large number of tunneling paths that do not interfere (there are  $2^M$  possible tunneling results, where  $M$  is the number of scatterers with free spin in the hopping act). Consequently the quantitative estimate of the effect of the free spins on interference magnetoresistance required. The present study provides such an estimate.

The role of interference effects in the magnetoresistance in the limit of long hops has been discussed (although without free spin consideration) in a number of publications [8–12]. However all these reports imply two important model assumptions which we do not believe to be obvious.

The first assumption is placing of scattering impurities in the nodes of some lattice. Usually one places scatterers in the nodes of a finite square lattice. The hopping impurities are on the diagonally opposite corners of the lattice. Then, one considers the shortest hopping paths that are along the lattice bonds (there are many such paths with equal length). In the present study we do not discuss the applicability of this assumption. However we do not use it.

The second assumption implies a neglect the variance of scatterer energies. In [4, 9, 10, 12] the authors consider scatterers with two possible energies  $W$  and  $-W$ , so that the variance of the absolute value of the scatterer energy was neglected. In [8, 11] the authors consider the flat distribution of scatterer energies at least as one of the possible models. However, then they assumed that the distribution of tunneling path amplitudes is the Gaussian one with the relative variance of the order of unity.

We argue that the second assumption is not correct for realistic semiconductors. Actually, the scatterer energies in semiconductors have flat distribution and this fact leads to the log-normal distribution of tunneling path amplitudes which is significantly different from a

normal one. This distribution plays a crucial role in our theory.

Let us consider some tunneling path. In accordance to the expression (4), its amplitude can be given as

$$J_i = I_{0,k_1} \prod_{k(i)}^{N_i} \frac{I_{k,k+1}}{E_k}. \quad (16)$$

Here  $J_i$  is the amplitude of  $i$ -th tunneling path. The index  $k(i)$  enumerates the scattering impurities that participate in the tunneling path  $i$ .  $N_i$  is a number of these impurities. Usually  $N_i \sim N/2$ , where  $N$  is the total number of scatterers in the resistor.  $k_1$  is the first of the impurities of this path. Index 0 ( $k = 0$ ) stands for the initial hopping impurity. The last index  $k = N_i + 1$  corresponds to the destination impurity of the hop.

Let us take a logarithm of the absolute value of  $J_i$  and write explicitly the term that is related to the hopping distance.

$$\ln \left| \frac{J_i}{I_0} \right| = -r_h/a + \sum_{k(i)} \mu_k^i, \quad \mu_k^i = -\frac{\Delta r_k}{a} + \ln \frac{I_0}{|E_k|} + \beta \ln \frac{r_{k-1,k} r_{k,k+1}}{r_{k-1,k+1} a}. \quad (17)$$

Here  $\Delta r_k$  is the additional distance an electron should tunnel due to scattering on the impurity  $k$ ,  $\Delta r_k = r_{k-1,k} + r_{k,k+1} - r_{k-1,k+1}$ .  $r_{i,j}$  is the distance between scattering impurities  $i$  and  $j$ , it may occur that some of this indices ( $i$  and  $j$ ) are equal to zero or to  $N_i + 1$  which means the starting impurity of the hop and the destination impurity, correspondingly.  $\beta$  is the pre-exponential factor in the overlap integral (see (3)).

We consider the scattering impurities to be situated in a thin strip to prevent the additional distance to be too large,  $\Delta r_k \leq a$ . Also, the typical values of scatterer energies are of the order of the Bohr energy, so  $I_0/|E_k| \sim 1$  (the sign of  $E_k$  is random). Finally, the third term is of the order of  $\ln r_{sc}/a$  where  $r_{sc}$  is the characteristic distance between scatterers in the resistor. In real situations this logarithm is not very large. Accordingly, we can consider  $\mu_k^i$  as a random value with expectation and variance of the order of unity.

Sometimes it is more useful to sum not over scatterers participating in the path  $i$ , but over all scatterers in the resistor. Therefore we introduce the modified values  $\tilde{\mu}_j^i$  that are equal to  $\mu_j^i$  if scatterer  $j$  participates in the given path and equals to 0 otherwise.

$$\tilde{\mu}_j^i = \begin{cases} \mu_j^i, & j \in \text{path } i \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

Again, we consider  $\tilde{\mu}_j^i$  as random quantities with math expectation  $E_\mu$  and variance  $D_\mu$  of the order of unity.

If one takes  $\tilde{\mu}_j^i$  as independent quantities (this assumption will be discussed later), one gets the normal distribution for  $\ln |J_i/I_0|$  (in accordance with the central limit theorem). The math expectation of  $\ln |J_i/I_0|$  is  $-r_h/a + NE_\mu$  and its variance is  $ND_\mu$ .

Hence the quantity  $|J_i|$  has a log-normal distribution law

$$f(|J_i|) = \frac{1}{|J_i|\sqrt{2\pi ND_\mu}} \exp \left[ -\frac{(\ln(|J_i|/I_0) + r_h/a - NE_\mu)^2}{2(ND_\mu)^2} \right], \quad |J_i| > 0. \quad (19)$$

The math expectation  $E(|J_i|)$  and variance  $D(|J_i|)$  are expressed as follows:

$$E(|J_i|) = I_0 \exp \left( -\frac{r_h}{a} + NE_\mu + \frac{ND_\mu}{2} \right); \quad (20)$$

$$D(|J_i|) = I_0^2 \exp \left( -\frac{2r_h}{a} + 2NE_\mu + 2ND_\mu \right). \quad (21)$$

This distribution law is significantly different from the one that can be obtained from normal distribution of  $J_i$ . Actually, it is the exponentially broad distribution. The important feature of the law (19) is that its math expectation (20) is exponentially larger than the value corresponding to the maximum of distribution density  $I_0 \exp(-r_h/a + NE_\mu)$ . It means that a sum of many tunneling path amplitudes (for example  $\tilde{I}_{12}$ ) should be dominated by exponentially small number of tunneling paths. In the next section we use this fact to derive a theory of interference magnetoresistance in the limit of long hops.

However, before we will discuss the magnetoresistance, let us consider the following moment. We derived the log-normal distribution (19) from the assumption that the quantities  $\tilde{\mu}_j^i$  where independent. Strictly speaking, this assumption is incorrect. The same impurities that form the interference pattern of the resistor participate in all tunneling paths (although in different combinations). Consequently we believe that it is reasonable to check the log-normal distribution by numerical calculations.

Let us take the resistor with 10 scattering impurities. This resistor has  $2^{10} = 1024$  tunneling paths, which is enough to reach the reliable statistics. The corresponding results for tunneling path amplitude distribution are shown on fig. 2. One can see that they are in a good agreement with log-normal law.

Another important property of the obtained tunneling path amplitude distribution is the fact that the sum of all tunneling paths is dominated by a small number of the summands. Let us in addition verify this result. In order to do this, we have to formalize the concept "dominate". We do this in the following way.

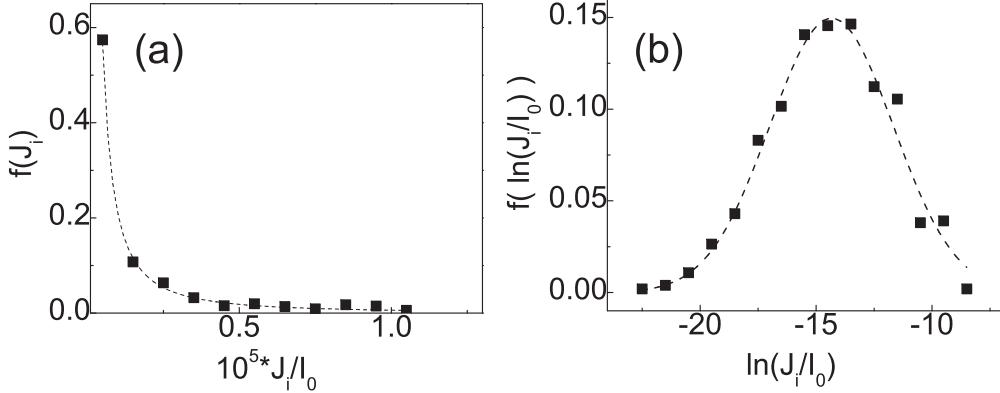


FIG. 2: Distribution functions of  $J_i$  (a) and  $\ln J_i/I_0$  (b) calculated for a tunneling path amplitudes in a single resistor compared to the log-normal distribution.

Consider the sum of all absolute values of tunneling path amplitudes  $\sum_i |J_i|$ . This sum has  $2^N$  summands. Let us now take some number  $N_{sign}$  of its largest summands, so that

$$\sum_k^{N_{sign}} |J_k| \geq 0.6 \sum_i |J_i|. \quad (22)$$

The left hand part of the inequality is the sum of  $N_{sign}$  largest  $|J_i|$ . The  $N_{sign}$  is considered to be the smallest number to satisfy inequality (22). We will specify  $N_{sign}$  tunneling paths with largest absolute values of amplitude as the significant paths.

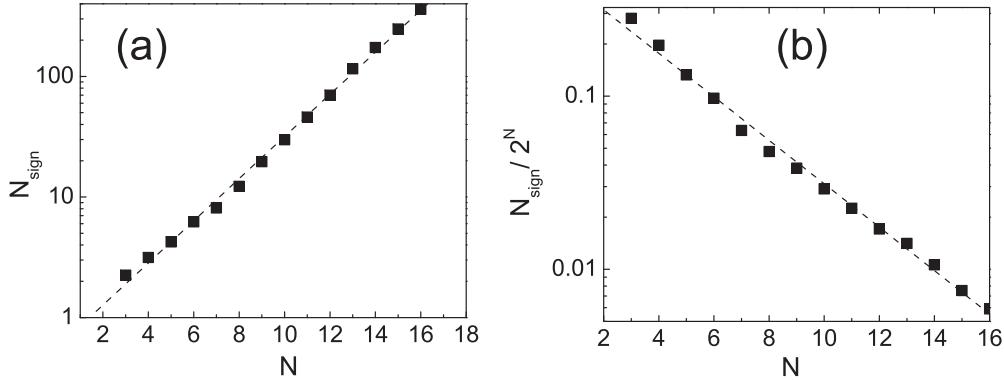


FIG. 3: (a) — dependence of  $N_{sign}$  on the scatterer number  $N$ . (b) — relative part of the significant paths  $N_{sign}/2^N$  versus scatterers number.

If the most of the tunneling paths are important within the sum  $\sum_i |J_i|$ , one should get  $N_{sign} \sim 0.5 \cdot 2^N$ . However if our assumption is correct and  $\sum_i |J_i|$  is controlled by a small number of its summands, we should have  $N_{sign} \ll 2^N$ . The corresponding numerical results

are shown on figure (3). One can see that, although the number  $N_{sign}$  exponentially grows with  $N$ , the relative part of significant paths exponentially decreases with  $N$ , demonstrating the validity of our assumption that the sum of contributions of different paths is controlled by a small number of summands. In the next section we will exploit it do derive the expression for magnetoresistance in the long hop limit.

## V. MAGNETORESISTANCE IN THE LIMIT OF LONG HOPS

In the spinless electron model the interference magnetoresistance is controlled by the overlap integral  $\tilde{I}_{12}$ , which is the sum of many tunneling path amplitudes  $J_i$ . We have shown that this sum is controlled by a relatively small number of the largest  $J_i$ . Now we consider the question which of  $J_i$  occurs to be large.  $J_i$  is proportional to the product  $\prod_j \exp(\mu_j^i)$  for impurities  $j$  participating in path  $i$ . The values  $\mu_j^i$  control whether the path  $i$  gives larger or smaller amplitude with an inclusion of the impurity  $j$ .

Strictly speaking,  $\mu_j^i$  is controlled not only by the properties of impurity  $j$  but also by other impurities participating in path  $i$ . However, we can estimate  $\mu_j^i$  for any scatterer  $j$  and a characteristic tunneling path.

$$\overline{\mu_j} = -\frac{y_j^2}{2r_{sc}a} + \ln \frac{I_0}{|E_j|} + \beta \ln \frac{r_{sc}}{2a}. \quad (23)$$

Here  $y_j$  is the distance between impurity  $j$  and the line connecting the starting and the final impurities of the hop,  $r_{sc}$  is the characteristic distance between the scatterers in the tunneling path.  $r_{sc}$  will be estimated later in our paper.

There are three opportunities for  $\overline{\mu_j}$ . First it may occur that  $\overline{\mu_j}$  is negative and has a large modulus, so  $\exp \overline{\mu_j} \ll 1$ . The corresponding impurities usually decrease the tunneling amplitude. The inclusion of a large number of such impurities makes the tunneling path amplitude exponentially small, so that this path will not be significant. Let us call such impurities with small  $\overline{\mu_j}$  as the irrelevant scatterers. Actually all impurities that are far from the line connecting the hopping impurities are irrelevant.

In contrast, it can occur that the absolute value  $|E_j|$  is much smaller than  $I_0$  and  $\exp \overline{\mu_j} \gg 1$ . Inclusion of such impurities makes the path amplitude larger. If some tunneling path does not include many such impurities, its amplitude becomes exponentially smaller than amplitudes of other paths (which include the corresponding impurities) and this path can

not be significant. So the impurities mentioned above should exist in most significant paths. We call these impurities as the backbone impurities.

Finally, there are impurities with  $\exp \bar{\mu}_j \sim 1$ . They can be included and can be excluded from a significant path. We call these impurities as the interference impurities.

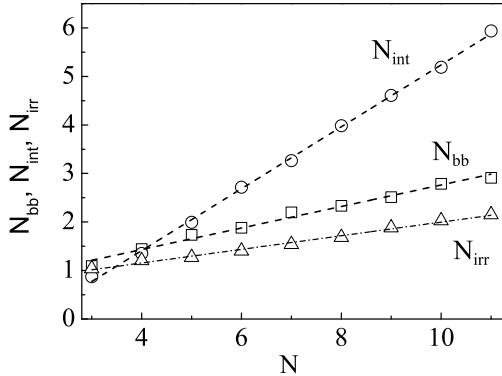


FIG. 4: Dependence of mean numbers of the different impurity types ( $N_{bb}$ ,  $N_{int}$  and  $N_{irr}$ ) on the total number of scatterers  $N$ . Each dot is averaged over 100 realizations.

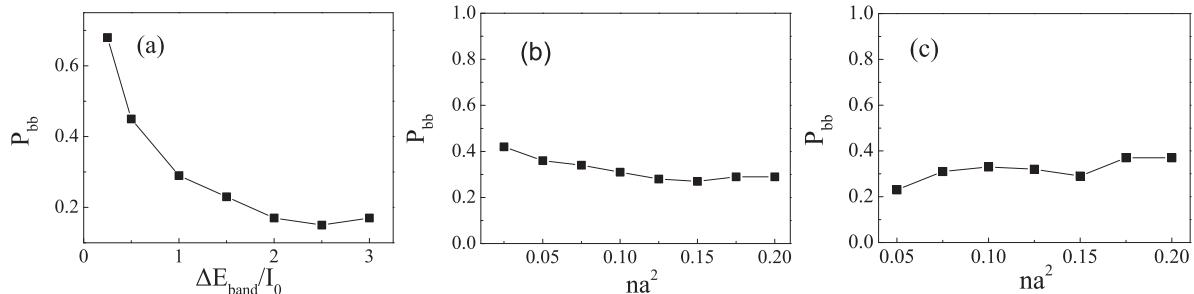


FIG. 5: Dependence of the relative number of backbone impurities  $P_{bb}$  on the system parameters. (a)— dependance on the width of impurity band  $\Delta E_{band}$  with  $na^2 = 0.2$  and pre-exponent factor  $\beta = 1$ , (b) — dependance on the impurity concentration with  $\Delta E_{band} = I_0$  and  $\beta = 1$ , (c) — dependance on the impurity concentration with  $\Delta E_{band} = I_0$  and  $\beta = -1$ .

Strictly speaking, the separation of impurities to irrelevant, backbone and interference ones is valid only for the exponentially broad distribution of scattering impurity energies (in this case  $\ln I_0/E_j$  is the leading term in (23) and can be large). However, even if the distribution is not exponentially broad, there are some impurities with small value of  $|E_j|$  that are good candidates to be the backbone impurities. Let us try to find such

impurities in numerical computations. To perform the computations, we consider impurity to be a backbone one when 90% of the significant paths include this impurity. Similarly, the irrelevant impurities are the ones that are not included in 90% of significant tunneling paths.

Figure 4 shows the averaged results for resistors with different length and with a constant width of the area occupied by the scatterers. One can see that the number of backbone impurities  $N_{bb}$  depends linearly on the scatterer number  $N$ . So we can estimate the relative number of backbone impurities in the long resistor limit. The same procedure can be followed for numbers of interference and irrelevant impurities  $N_{int}$  and  $N_{irr}$ . Note that the relative parts of different types of impurities can depend on the width of the area exploited for calculations. For very wide area most impurities will be irrelevant. So we redefine the relative number of backbone impurities to make it model-independent

$$P_{bb} = \lim_{N \rightarrow \infty} \frac{N_{bb}}{N_{bb} + N_{int}}. \quad (24)$$

$P_{bb}$  is the relative part of backbone impurities with irrelevant impurities excluded. It does not depend on the width of the model area if it is large enough. Fig. 5 shows our computation of the dependence of  $P_{bb}$  on the impurity band width  $\Delta E_{band}$ , concentration  $n$  and pre-exponent factor  $\beta$ . One can see that  $P_{bb}$  increases with the decrease of  $\Delta E_{band}$ . For large band width  $P_{bb}$  should tend to some finite value as the impurities with large energy are always irrelevant scatterers.  $P_{bb}$  weakly depend on concentration and pre-exponent factor at least in observed range of parameters.

Now let us make use of the existence of backbone impurities to calculate the magnetoresistance in the long hops regime. In the present study we restrict ourselves with the simplest model of independent links. This model assumes that any two interference impurities are always separated with a backbone one. Therefore this model should be valid for  $P_{bb} > 0.5$ . However the results of our computations agrees with this model even for smaller  $P_{bb}$ .

Let us illustrate our considerations with the following model resistor (see fig. 6). We consider, first, the spinless electron model. This resistor contains two backbone impurities, two interference impurities and two irrelevant ones. So the total number of scatterers is  $N = 6$ , the number of tunneling paths is  $2^N = 64$ . However, in our model we assume that one can restrict himself with consideration of significant paths — ones that contain all backbone scatterers and bypass all irrelevant impurities. Therefore instead of 64 tunneling

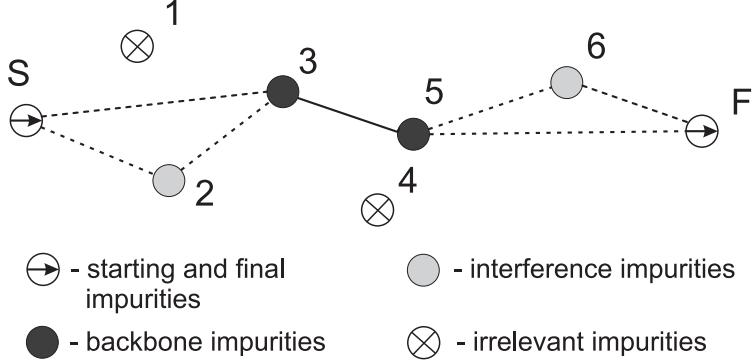


FIG. 6: The example of the hopping resistor with six scatterers. The scattering impurities are numerated while the starting and final impurities in this scatterer are marked with indexes  $S$  and  $F$  correspondingly.

paths one can consider only four ones:  $(S \rightarrow 3 \rightarrow 5 \rightarrow F)$ ,  $(S \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow F)$ ,  $(S \rightarrow 3 \rightarrow 5 \rightarrow 6 \rightarrow F)$  and  $(S \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6 \rightarrow F)$ .

The amplitudes of these paths are expressed by Eq. (17). One can see that all amplitudes of these paths contain the term  $I_{35}/E_3 E_5$ . It corresponds to backbone impurities. Then, all paths contain connections between the backbone parts. For example, impurities  $S$  and 3 can be connected with either  $I_{S3}$  or  $I_{S3}I_{23}/E_2$ . One can show that within our approximations these connections are independent, i.e. the net tunneling amplitude  $J = \sum_i J_i$  can be expanded as

$$J = \left( I_{S3} + \frac{I_{S2}I_{23}}{E_2} \right) \frac{I_{35}}{E_3 E_5} \left( I_{5F} + \frac{I_{56}I_{6F}}{E_6} \right). \quad (25)$$

The backbone impurities effectively separate the resistor into independent parts — the interference links. The amplitude of each link can be described in the same way as a tunneling amplitude for the resistor with one scatterer. The net tunneling amplitude of the resistor in the independent links model is the product of terms corresponding to all links and additional terms corresponding to backbone impurities.

$$J = \prod_{\text{backbone}} \frac{I_{kj}}{E_j} \cdot \prod_{\text{interference}} \left( I_{i1,i3} + \frac{I_{i1,i2}I_{i2,i3}}{E_{i2}} \right). \quad (26)$$

Here the first term is the product of  $1/E_j$  for all backbone impurities  $j$  and the product of all the overlap integrals  $I_{kj}$  between neighbouring backbone impurities  $k$  and  $j$  (there should be no interference link between these impurities). The second product is over interference links. We assume that each link  $i$  contains starting and final impurities  $i1$  and  $i3$ , correspondingly,

(that are the backbone impurities) and intermediate interference impurity  $i2$ . For example in our model resistor for the link connecting impurities  $S$  and 3 the index  $i1$  corresponds to the starting impurity  $S$ . The impurities  $i2$  and  $i3$  are actually the scatterers 2 and 3 correspondingly. The expression (26) is the generalization of (25) for the arbitrary resistor.

From Eq. (26) one can easily obtain the expression for magnetoresistance corresponding to long hops and spinless electrons in our model of independent links.

$$\ln \frac{R(H)}{R(0)} = - \left\langle \sum_i \ln \frac{|J_{i1}(H) + J_{i2}(H)|^2}{|J_{1i}(0) + J_{2i}(0)|^2} \right\rangle \approx -\bar{N}_{int} \left\langle \ln \frac{|J_{i1}(H) + J_{i2}(H)|^2}{|J_{1i}(0) + J_{2i}(0)|^2} \right\rangle_{int}, \quad (27)$$

where

$$J_{i1}(H) = I_{i1,i3}(H), \quad J_{i2} = \frac{I_{i1,2}(H)I_{i2,i3}(H)}{E_{i2}}. \quad (28)$$

Here index  $i$  numerates interference links;  $I_{i1,i2}$ ,  $I_{i1,i3}$  and  $I_{i2,i3}$  depend on the magnetic field through the phase  $\varphi$ .  $\bar{N}_{int}$  on the right hand side is the mean number of interference links between the hopping impurities. It is proportional to the characteristic scatterer number  $N$ . Angle brackets with index  $int$  mean that the averaging is over interference links (that correspond to interference scatterers in a resistor).

$N$  is controlled by the area which does not give a significant addition to the tunneling exponent. Due to the presence of backbone impurities one should take the area with the constant width (independent on hoping distance)  $\rho \sim \sqrt{r_{sc}a} \ll \sqrt{r_ha}$ . The width  $\rho$  and the distance between neighbor impurities can be expressed as

$$\rho \sim \left(\frac{a}{n}\right)^{1/(d+1)}, \quad r_{sc} \sim \left(\frac{a^{(1-d)/2}}{n}\right)^{2/(1+d)}. \quad (29)$$

Accordingly, the mean number of scatterers  $N = nr_h\rho^{d-1}$  is proportional to the hopping length  $r_h$ .

Finally, let us note that the quantity under the logarithm in (27) can significantly differ from unity only when amplitudes  $J_{i1}$  and  $J_{i2}$  are comparable (otherwise one of the amplitudes dominates and the discussed quantity is  $\approx 1$ ). So we can substitute the averaging over interference links by averaging over all scatterers and, correspondingly, substitute  $\bar{N}_{int}$  by  $N$ .

$$\ln \frac{R(H)}{R(0)} \approx -N \left\langle \ln \frac{|J_{i1}(H) + J_{i2}(H)|^2}{|J_{1i}(0) + J_{2i}(0)|^2} \right\rangle. \quad (30)$$

Here averaging is over all scattering impurities  $i$ . In the explicit expressions for  $J_{i1}$  and  $J_{i2}$  (28) one should substitute impurity  $i2$  by impurity  $i$ . The impurities  $i1$  and  $i3$  should be

substituted with the backbone impurities neighboring to  $i$ . With this expression the problem of magnetoresistance in the case of long hops is deduced to the magnetoresistance in the case of short hops for resistors with  $r_h \sim 2r_{sc}$ .

The expression (30) comes from the approximation of the sum of all tunneling paths  $\sum J_i$  and thus does not include free spins. To include free spins, one should derive the expression for magnetoresistance from (12) with the approximation of  $N_{bb} \gg N_{int}$ . The corresponding expression for  $\Gamma_{12}$  is derived in the Appendix A (eq. A3). With above mentioned arguments one can get from (A3) the following expression for magnetoresistance for long hops that includes the contribution of free spins:

$$\begin{aligned} \ln \frac{R(H)}{R(0)} = & -(1 - P_{free})N \left\langle \ln \frac{|J_{i1}(H) + J_{i2}(0)|^2}{|J_{i1}(0) + J_{i2}(0)|^2} \right\rangle - \\ & -P_{free}N \left\langle \ln \frac{P_{\uparrow\uparrow}|J_{i1}(H) + J_{i2}(H)|^2 + (1 - P_{\uparrow\uparrow})(|J_{i1}(H)|^2 + |J_{i2}(H)|^2)}{|J_{i1}(0) + J_{i2}(0)|^2/2 + (|J_{i1}(0)|^2 + |J_{i2}(0)|^2)/2} \right\rangle \end{aligned} \quad (31)$$

One can see that terms in angle brackets in (31) have the same form as the magnetoresistance in the case of short hops (compare with expr. (8) and (9)). This magnetoresistance is discussed in section III. However the "length" of the "resistors" corresponding to this terms in (31) is controlled by the distance  $r_{sc}$  between neighbor scatterers in a resistor rather than by the actual hopping distance  $r_h$ . Note that  $r_{sc}$  is still larger than the distance between neighboring impurities  $n^{-1/d}$ .

The expression (31) means that all properties discussed for the short hops in section III exist also in the case of long hops (at least as long as the independent link approximation remains applicable). There is a negative linear magnetoresistance due to the suppression of the interference by the magnetic field

$$\ln \frac{R(H)}{R(0)} \propto -N(1 - P_{free})r_{sc}^{2+d/2}H \propto -r_h H. \quad (32)$$

One can see that the dependence of the magnetoresistance on the hopping distance  $r_h$  (and, correspondingly, on temperature) becomes weaker than for the short hop case (compare  $r_h$  to  $r_h^{2+d/2}$ ). At large temperatures there is additional temperature dependence due to a decrease of  $P_{free}$  with decreasing temperature. However, in a common situation when most spins are frozen and  $P_{free} \ll 1$ , one can neglect the dependence  $P_{free}(T)$  and assume that  $1 - P_{free} \approx 1$ . We argue, therefore, that the interference magnetoresistance mechanism is

not affected by small concentrations of impurities with free electron spin despite the fact that these impurities in the limit of low temperatures are included in the hopping process.

Let us now discuss the saturation field for this negative magnetoresistance. The conventional theory which considers the destructive interference related to all tunneling paths leads to the saturation field  $H_{sat} \sim \Phi_0/r_h\rho$ . It is the field that suppresses coherence in a characteristic pair of tunneling paths. Note that  $H_{sat}$  tends to zero with decreasing temperature. However, in our approximation for long hops one considers only significant paths that in some sense are similar to each other. Moreover, the interference phenomena are localized in the relatively small interference links. Accordingly, the saturation field is controlled by suppression of interference inside one link. It leads to  $H_{sat} \sim \Phi_0/r_{sc}\rho$ . This field is independent on temperature and on hopping distance.

There also exists the positive magnetoresistance due to spin alignment by the magnetic field, which is quadratic in  $H$  and has a strong temperature dependence. It arises from the term in (31) related to the free spins and its expression is

$$\ln \frac{R(H)}{R(0)} \propto NP_{free}(nr_{sc}^{(d+1)/2}a^{(d-1)/2}) \left(\frac{\mu_b g H}{T}\right)^2 \propto r_h P_{free} T^{-2} H^2. \quad (33)$$

Again, this magnetoresistance mechanism has a strong temperature dependence. If the hopping is over the Coulomb gap states, the magnetoresistance is  $\propto P_{free} T^{-2.5}$  where  $P_{free}$  logarithmically decreases with temperature. The temperature dependence of this mechanism is stronger than the one for the other known positive magnetoresistance mechanisms including the wave-shrinkage magnetoresistance and the magnetoresistance due to the suppression of hops between the impurities with different occupation numbers [23, 24]. However, for the standard semiconductor with most electron spins frozen the value  $P_{free}$  is small, so this magnetoresistance mechanism becomes important only at low temperatures. Also one should note that this mechanism saturates at magnetic field of the order of  $T/\mu_b g$  that is also small at low temperatures.

### A. numeric computation

In our analytical theory we have used the approximation of large variance of scatterer energy. More exactly, only when the variance  $D_E$  of  $\ln I_0/|E|$  (where  $E$  is impurity energy measured from Fermi level) is large, the model of interference links becomes justified. Only

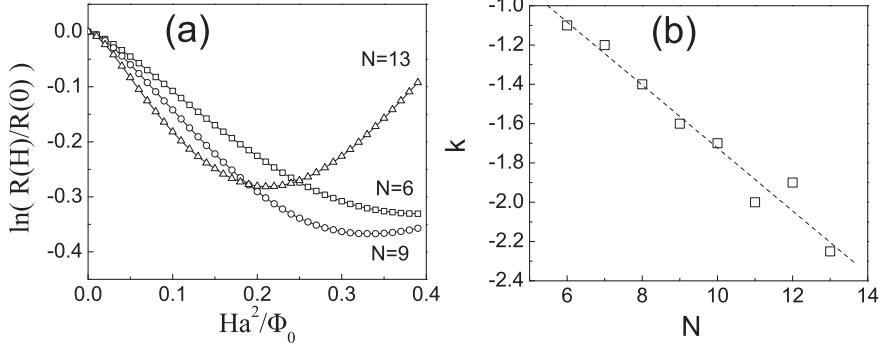


FIG. 7: (a) — interference contribution to magnetoresistance computed with eq. (12) for different numbers of scatterers  $N \propto r_h$ ; (b) — dependance of linear expansion coefficient  $k$  on the scatterer number  $N$ . The probability of an electron spin to be free was considered to be  $P_{free} = 0.75$ .

in this case one can discriminate between backbone, irrelevant and interference impurities and consider the number of interference impurities to be small.

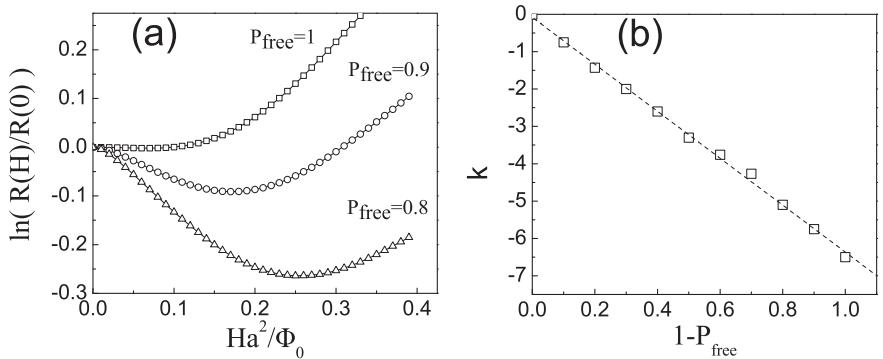


FIG. 8: Interference contribution to magnetoresistance computed with eq. (12) for different probabilities  $P_{free}$  (a) and dependance of linear expansion coefficient  $k$  on  $1 - P_{free}$  (b). The calculation was made for resistors with  $N = 10$ .

In real semiconductors the discussed variance is of the order of unity. However, some of our results are expected to be valid for real semiconductors. For example, the log-normal distribution of tunneling path amplitudes remains correct for any  $D_E \neq 0$ . Also we have shown numerically that some other results (the existence of significant paths and backbone impurities) can be applied for realistic distribution of scatterer energies. Consequently we should also test our final results (32) and (33) with the numeric computations.

In our computation we simulate the random distribution of scatterers in the area between the hopping impurities. We consider impurities only in the thin area around the hopping

line with the width  $\rho \propto \sqrt{r_{sc}a}$ . The energies of the scatterers are randomly distributed with constant density within the interval  $-I_0 < E_j < I_0$ . Note that this distribution corresponds to rather small  $P_{bb}$ . However we show that even for this distribution numerical results agree with our model.

We consider the 2D distribution of the impurities. In our computation we keep the impurity concentration constant ( $na^2 = 0.2$ ), so that the number of scatterers is directly proportional to the hopping distance  $r_h$ . We assumed the hopping over the Coulomb gap states, so that  $\mu_b g/T \propto r_h^2$ .

To consider the system with free electron spins, we calculate all the tunneling path amplitudes  $J_i$  and estimate the magnetoresistance with a help of Eq. (12). We performed the averaging over 1000 realizations (i.e. 1000 different critical hopping resistors). The results of these simulations are presented in figs. 7 and 8. We do not use any of our approximations (essential paths, backbone scatterers) in our computations. Instead we include all intermediate impurities and tunneling paths into the computation process.

One can see that numerical results agree with analytic expressions (32) and (33), i.e. there is observable linear negative magnetoresistance (even for large  $P_{free}$ ); the linear expansion coefficient  $k$  of magnetoresistance is proportional to  $r_h \propto N$  and  $1 - P_{free}$ . Also there is a positive quadratic magnetoresistance that increases with  $P_{free}$  and increases with  $N$  stronger than the linear magnetoresistance.

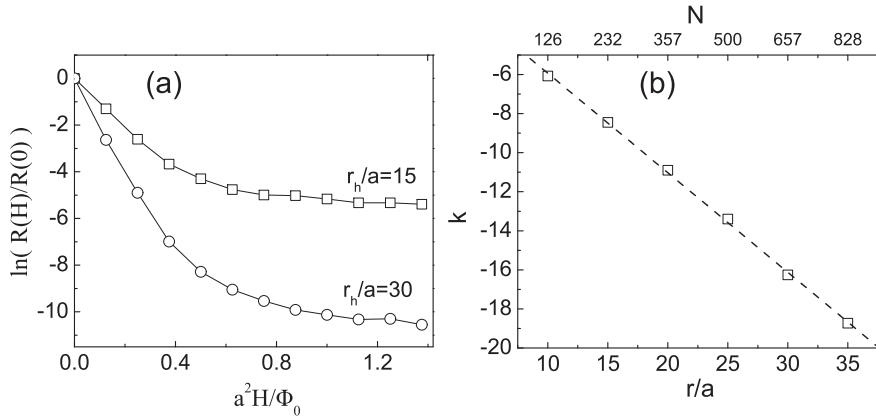


FIG. 9: The magnetoresistance for large hopping distances  $r_h$  and  $P_{free} = 0$ . (a) — magnetoresistance for two different  $r_h$ . (b) — dependance of linear magnetoresistance on  $r_h$  and  $N$ . One can see that dependance  $k \propto r_h$  stays justified when the number of scatterers grows faster than  $r_h$ .

However the question remains whether the computed linear dependence  $k(r_h)$  is the real

semiconductor property or it is due to the model choice (in our computations we have chosen the scatterers in the strip of constant width, and the number of scatterers was  $N \propto r_h$ ). We can not consider a larger number of scatterers and compute all tunneling amplitudes  $J_i$  because of the technical reason (the number of these amplitudes grow exponentially with  $N$ ). However we can bypass this problem when we consider the system without free spins. In this case we can use the summation algorithm that gives the correct result (in terms of expression (12)) with the computation time  $\propto N^2$ .

Accordingly we calculated the magnetoresistance for the case of  $P_{free} = 0$  and large  $r_h/a \leq 35$ . We also took scatterers in a much wider area with the width  $\rho \propto \sqrt{r_h}$  (so  $N \propto r_h^{3/2}$ ). This area contains all scatterers that can be included in tunneling path without making it exponentially small (independent on backbone impurities). Figure 9 shows the corresponding results. One can see that the linear magnetoresistance is proportional to  $r_h$  and not to  $N$ . Finally this calculation shows (fig. 9 (a)) no signs of decrease of saturation magnetic field with increasing hopping distance. This fact gives additional support to the model of interference links.

## VI. COMPARISON WITH EXPERIMENT

In this section we discuss one experimental result that does not agree with the conventional picture of hopping negative magnetoresistance. These experiments demonstrate the suppression of negative magnetoresistance at low temperature observed in [18, 19]. In the corresponding experiments the magnetoresistance was measured at different temperatures in 2D GaAs-AlGaAs heterostructures, where both the wells and the barriers were doped by acceptor impurity Be and, thus, the acceptors within the well included double occupied  $A^+$  centers. In contrast to the predictions of the conventional theory [1, 4, 7], it has been observed that the negative magnetoresistance is suppressed at low temperatures.

It worth noting that the similar phenomenon has been observed earlier in bulk semiconductors [14, 15]. It has been explained with the assumption of non-Coulomb character of impurity potential at large distances and effect of the Coulomb gap. However, as explicitly shown in [19], this explanation does is not valid for 2D systems (due to different asymptotic of the wavefunctions of localized electrons in 2D with respect to 3D one). So the spin alignment magnetoresistance mechanism has been invoked to explain this phenomenon in 2D.

However no detailed theory of this magnetoresistance mechanism has been proposed.

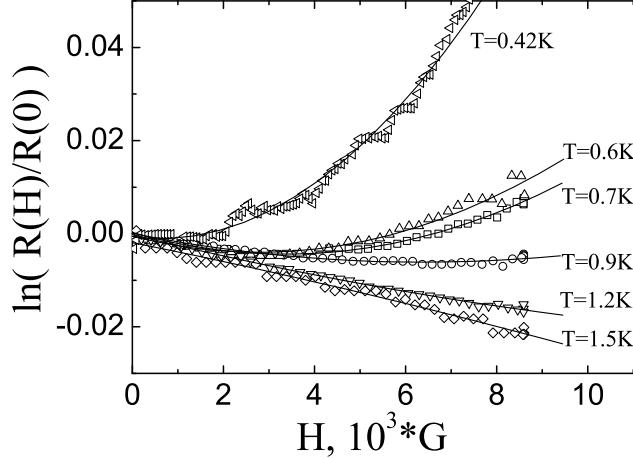


FIG. 10: Magnetoresistance of 2D semiconductor structures at different temperatures. The experiment results (dots) from [19] are compared with parabolic law (34) (lines).

It is important to note that the systems observed in [18, 19] include scatterers of two types (due to the double occupation) — the first type is the acceptor in the quantum well that has high activation energy and a short localization radius. The second type corresponds to the double occupied acceptors in the well [25, 26] and the localized states of the hole in the well bound to the barrier acceptors. These states have relatively small activation energy and a large localization radius. It is likely that the spins of holes on single occupied acceptors in the well are free, while the spins of holes on the scatterers of the second type are frozen. Thus, in the considered system one has noticeable values of both  $P_{free}$  and  $1 - P_{free}$ , and therefore both negative linear magnetoresistance and positive spin ordering magnetoresistance should be present in the system.

Figure 10 shows these experimental data compared with the parabolic law

$$\frac{\Delta R}{R} \approx \ln \frac{R(H)}{R(0)} = -kH + k_2 H^2, \quad (34)$$

which is simply the first two terms of the magnetoresistance expansion. One can see that the experimental data are in a good agreement with (34). It means that the higher terms of expansion can be neglected when discussing these experiments.

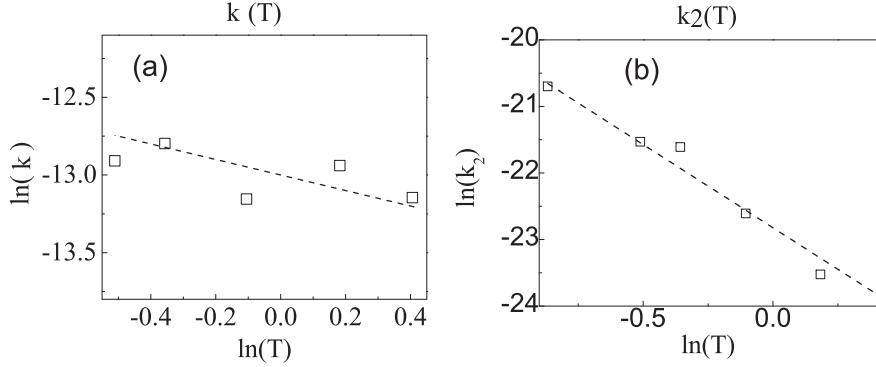


FIG. 11: The temperature dependence of  $k$  (a) and  $k_2$  (b) (dots) from [19] compared to the theoretical results (32) and (33) (lines).

The next step is to compare temperature dependencies of  $k$  and  $k_2$  with theoretical predictions. Assuming that the conduction is controlled by the Coulomb gap states the conventional theory predicts  $k \propto T^{-3/2}$  and  $k_2 \propto T^{-3/2}$  due to wavefunction shrinkage magnetoresistance mechanism. These dependencies lead to the increase of the role of negative magnetoresistance at low temperatures. The extremal (minimal) magnetoresistance value increases with decreasing temperature as  $\propto T^{-3/2}$ . Our theory (32,33) for long hops gives  $k \propto T^{-1/2}$ , also we predict the strong dependence  $k_2 \propto T^{-5/2}$  due to spin alignment magnetoresistance. The value of the magnetoresistance minimum depends on temperature as  $T^{3/2}$ .

Figure 11 compares experimental data with our results (32,33) demonstrating a good agreement with our predictions. The comparison with the conventional theory results in a significantly worse fit. So we argue that our theory is at least in semiquantitative agreement with experiment.

## VII. DISCUSSION

As we have mentioned earlier, the conventional considerations of the interference mechanism of magnetoresistance in the long hop limit [8–12] neglect one important factor. It is the variance of logarithm of scatterer energy  $D_E$ . In the preceding studies this parameter has been set to be  $D_E = 0$  whether explicitly or implicitly. We started our theory from the opposite limit  $D_E \gg 1$ . Actually, when  $D_E = \infty$ , there is no interference magnetoresistance as the single tunneling path dominates all the hopping process. Our theory can be

considered as a first correction related to a finite  $D_E$ . One can calculate further terms by considering sequences of two interference impurities separated by a backbone ones, then of three interference impurities and so on.

We did not intend to go this far in the present study. However, we presume that the proportionality of magnetoresistance to  $r_h$  should exist also for the higher terms. Actually, for any  $D_E \neq 0$  there is some probability to find a backbone impurity — the inclusion of this impurity significantly increase the tunneling amplitude. So the resistor in the long hop limit is still separated into the parts with this backbone impurities although these parts can be large for small (but finite)  $D_E$ . The magnetoresistance should be proportional to the number of these parts and thus should be proportional to  $r_h$ . The dependence on  $P_{free}$  can be different in the more rigorous theory as the parts separated by the backbone impurities can contain more than one free spin.

One question is widely discussed in the physics of hopping with interference in the long hop limit. When the hops are short, the net tunneling amplitude is usually positive. It is not clear, however, whether the sign of net tunneling amplitude is completely random in the long hop limit or whether a positive sign still prevails. Our theory gives the following answer for this question. The sign is completely random when the resistor contains at least one backbone impurity. Naturally the net tunneling amplitude is controlled by significant tunneling paths that include this backbone impurity. The sign of the energy of this impurity is random, so the sign of the net tunneling amplitude is also completely random. Clearly, at any temperature (and hopping distance  $r_h$ ) there is some chance to find a resistor without backbone impurities but the probability of such an event drops exponentially with  $r_h$ . Therefore the average sign of net tunneling amplitude also drops with  $r_h$  exponentially.

Summing up, we generalized the theory of interference effects in the hopping magnetoresistance to include the contribution of scatterers with free electron spins. We considered both the case of short hops when the mean number of scatterers in a resistor is less than unity (many resistors in this case have no scatterers and thus no interference effects) and the case of long hops when the mean number of scatterers in a resistor is large. For the case of long hops we developed a new approach to the problem of interference magnetoresistance that is based on assumption of large variance of the logarithm of scatter energy  $D_E$  (the conventional approaches consider  $D_E = 0$ ). We showed that our approach is in a good agreement with numerical computations with realistic value of  $D_E$ . Our theory allows one

to calculate explicitly the temperature dependence of magnetoresistance and its dependence on the relative ratio of free spins  $P_{free}$  for the short hops and the long hops cases. Our results are in semiquantitative agreement with experimental data on magnetoresistance in  $GaAs - AlGaAs$  2D structures.

We are grateful to A.S. Ioselevich and N.S. Averkiev for many fruitful discussions. Our work was supported by RFBR foundation (project n. 10-02-00544). A.V.S. also acknowledges support from "dynasty" foundation and from the program of RAS presidium "the support for young scientists".

### **Appendix A: The expression for $\Gamma_{12}$ with the assumption of large number of backbone scatterers and with an account of free spins.**

To consider the problem of magnetoresistance in a system with free spins, we should start from the expression (10) for the hopping rate. We also adopt the model of independent links, i.e. we assume that  $\Gamma_{12}$  is controlled by the significant tunneling paths that differ from each other only in local interference links.

First, we note that inclusion or exclusion of a scatterer with frozen spin does not change the tunneling result independently of spin configuration. Each significant tunneling path  $j$  has a pair that differs from  $j$  only by the interference link  $i$  (for any link  $i$ ). One can see that if link  $i$  does not contain a free spin, then by combining such pairs one can take out the factor

$$\left| I_{i1,i3} + \frac{I_{i1,i2}I_{i2,i3}}{E_{i2}} \right|^2 \quad (A1)$$

corresponding to this interference link  $i$ .

Also one can take out all factors corresponding to backbone scatterers. However, these factors do not contribute to interference magnetoresistance, so we omit them

$$\Gamma_{12} \propto \prod_{i \in \text{frozen}} \left| I_{i1,i3} + \frac{I_{i1,i2}I_{i2,i3}}{E_{i2}} \right|^2 \cdot \sum_c P(c) \sum_{res} \left| \sum_{j(c,res)} \prod_{s \in \text{free}} L_s^{(j)} \right|^2. \quad (A2)$$

Here the first product is taken over interference links with frozen spin or interference links with unoccupied impurities. The product over  $s$  is taken over interference scatterers with free spins.  $L_s^{(j)}$  is the amplitude of the interference link  $s$  in the path  $j$ . It can be equal to either  $I_{s1,s3}$  if the path  $j$  does not include scatterer  $s$  or to  $I_{s1,s2}I_{s2,s3}/E_{s2}$  if the path  $j$  includes  $s$ .

Further, we use the approximation  $N_{bb} \gg N_{int}$  once again. We consider that for any interference link  $s$  with free spin the previous impurity with free spin  $s-$  is a backbone one. Accordingly the next impurity with free spin  $s+$  (for any interference link with free spin  $s$ ) is also a backbone impurity.

With this assumption one can see that two paths that differ only by a single impurity with free spin  $s$  lead to the same tunneling result for all configurations when spin projections on  $s$  and  $s-$  are the same. In other spin configurations this tunneling paths lead to different results, however these results differ only by the replacement of final projections of spin on the impurities  $s$  and  $s+$ . Note that in this model the inclusion or exclusion of an interference impurity  $s$  does not change the interference pattern on the other interference impurities with free spin that follow  $s$ . This interference is controlled by initial spin projections on  $s+$  and on other backbone impurities with free spins. Thus one can obtain the following expression for  $\Gamma_{12}$ .

$$\begin{aligned} \Gamma_{12} \propto & \prod_{i \in \text{frozen}} \left| I_{i1,i3} + \frac{I_{i1,i2}I_{i2,i3}}{E_{i2}} \right|^2 \times \\ & \times \prod_{s \in \text{free}} \left[ P_{\uparrow\uparrow}^{s-,s} \left| I_{s1,s3} + \frac{I_{s1,s2}I_{s2,s3}}{E_{s2}} \right|^2 + P_{\uparrow\downarrow}^{s-,s} \left( |I_{s1,s3}|^2 + \left| \frac{I_{s1,s2}I_{s2,s3}}{E_{s2}} \right|^2 \right) \right]. \end{aligned} \quad (\text{A3})$$

Here  $P_{\uparrow\uparrow}^{s-,s}$  is the probability for spins on impurities  $s$  and  $s-$  to have the same projection. Naturally, this probability does not depend on  $s$ :  $P_{\uparrow\uparrow}^{s-,s} = P_{\uparrow\uparrow}$  with  $P_{\uparrow\uparrow}$  defined in (7). Similarly,  $P_{\uparrow\downarrow}^{s-,s} = 1 - P_{\uparrow\uparrow}$  for any interference link with free spin  $s$ .

- [1] B.I. Shklovskii and A.L. Efros, "Electronic Properties of Doped Semiconductors" (Springer, Berlin, 1984).
- [2] Sladek R.J., J. Phys. Chem. Sol., **5**, 157 (1958)
- [3] V.L. Nguyen, B.Z. Spivak, and B.I. Shklovskii, Pisma Zh. Eksp. Teor. Fiz. **41**, 35 (1985); Zh. Eksp. Teor. Fiz. **89**, 1770 (1985) [Sov. Phys. JETP **62**, 1021 (1985)].
- [4] Shklovskii B.I., Spivak B.Z. In: Hopping transport in solids, ed. by M.Pollak and B.Shklovskii, Elsevier, 1991, p. 271
- [5] H.L.Zhao, B.Z.Spivak, M.P.Gelfand, S.Feng Phys. Rev. B, **44**, 10760 (1991)
- [6] W.Schirmacher Phys. Rev B, **41**, 2461 (1990)
- [7] M.E. Raikh, G.F. Wessels, Phys. Rev. B, **47**, 15609 (1993)

- [8] U.Sivan, O.Entin-Wohlman, Y.Imry, PRL, **60**, 1566 (1988)
- [9] Eduardo G. Aponte, Ernesto Medina, PRE, **58**, 4246 (1998)
- [10] Yeong-Lieh Lin and Franco Nori, PRL, **76**, 4580 (1996)
- [11] Yeong-Lieh Lin and Franco Nori, PRB **53**, 15543 (1996)
- [12] Yeong-Lieh Lin, PRB, **58** 13544 (1998)
- [13] R. Rentzsch, A. N. Ionov, B. Sandow, P. Stefanov, P. Fozooni and M. L. Lea, Phys. Status Solidi B **203**, 487 (1997)
- [14] N.V.Agrinskaya, V.I.Kozub, D.V.Shamshur Zh.Eksp.Teor. Fiz., **107**, 2063 (1995)
- [15] N.V.Agrinskaya, V.I.Kozub, R.Rentzch, M.D.Lea, P.Fozoni . Zh. Eksp. Teor. Fiz., **111**, 1477-1493 (1997)
- [16] N.V.Agrinskaya, V.I.Kozub, Yu.L.Ivanov, V.M.Ustinov, A.V.Chernyaev, D.V.Shamshur. JETP, **93**, 424 (2001)
- [17] N.V.Agrinskaya, V.I.Kozub, V.M.Ustinov, A.V.Chernyaev, D.V.Shamshur JETP Letters **76**, 360 (2002)
- [18] N.V.Agrinskaya, V.I.Kozub, D.V. Poloskin, A.V.Chernyaev, D.V.Shamshur JETP Letters **80**, 30 (2004)
- [19] N.V. Agrinskaya, V.I. Kozub, A.V. Shumilin, and E. Sobko, Phys. Rev. B, **82** 075201 (2010)
- [20] A. Miller, E. Abrahams, Phys.Rev., **120**, 745 (1960)
- [21] R.N. Bhatt, P.A. Lee, Phys. Rev. Lett., **48**, 344 (1982)
- [22] J.R. Eggert, Phys. Rev. B, **29**, 6664 (1984)
- [23] A.Kurobe, H.J.Kamimura Phys. Soc. Jap., **51**, 1904 (1982)
- [24] K.A. Matveev, L.I. Glazman, Penny Clarke, D. Ephron, M.R. Beasley, Phys. Rev. B 52, 5289 (1995).
- [25] D.M. Larsen, Phys. Rev B., **47**, 16333 (1993).
- [26] N.S. Averkiev, A.V. Rodina, Phys. Solid State, **35**(4), 538 (1993)